Summary of Lecture 4

Elevator Control Power and Trim

1. How to achieve balance at different flight conditions ($C_L$ or $\alpha$)? Elevator Deflection.

2. Effect of flap or control surface: $C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_\delta} \delta$ where $C_{L_\delta} = C_{L_\alpha} \tau$. Thin airfoil theory states that

$$\tau = \frac{2}{\pi} \left[ \sqrt{\lambda_f(1-\lambda_f)} + \sin^{-1} \sqrt{\lambda_f} \right]$$

and

$$\frac{\partial C_{Mac}}{\partial \delta} = -\sqrt{\lambda_f(1-\lambda_f)}^3$$

where $\lambda_f$ is the ratio between the flap chord to the airfoil chord. For three dimensional applications, $\tau$ can be obtained from Fig. 2.21, pp. 64 in Nelson’s [1], which is shown here in Fig. 1

Figure 1: Effect of a control surface deflection.

3. Elevator Effect:

$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_\delta} \delta_e$$

and

$$C_{M_{CG}} = C_{M_{CG}0} + C_{M_{CG\alpha}} \alpha + C_{M_{CG\delta}} \delta_e$$

where

$$C_{L_\delta} = \frac{\eta}{S} \frac{S_t}{C_{L_{at}}} \tau_e$$

and Elevator Control Power: $C_{M_{CG\delta}} = -\eta V H C_{L_{at}} \tau_e$

4. Trim: Given airplane weight $W$ and geometrical characteristics (e.g., $S$), cruise speed $V$, and altitude $h$. Then, $W = \frac{1}{2} \rho V^2 S C_L \Rightarrow C_L$. Then, determining the airplane characteristics $C_{L_0}, C_{L_\alpha}, C_{L_\delta}, C_{M_{CG0}}, C_{M_{CG\alpha}},$ and $C_{M_{CG\delta}}$, and write the trim equations

$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_\delta} \delta_e$$

and

$$0 = C_{M_{CG0}} + C_{M_{CG\alpha}} \alpha + C_{M_{CG\delta}} \delta_e \Rightarrow \alpha, \delta_e$$
These two equations give

\[ \delta_{\text{trim}} = -\frac{C_{M_{0 CG}} C_{L_{0}} + C_{M_{a CG}} (C_{L_{\text{trim}}} - C_{L_{0}}) \ C_{M_{e CG}} C_{L_{e}}}{C_{L_{\text{trim}}} - C_{L_{0}} - C_{L_{a}} \delta_{\text{trim}}} \]

and

\[ \alpha_{\text{trim}} = \frac{(C_{L_{\text{trim}}} - C_{L_{0}}) - C_{L_{a}} \delta_{\text{trim}}}{C_{L_{a}}} \]

5. Problem 2.1 in Nelson [1], pp. 85: If the slope of the \( C_{M} - C_{L} \) curve is -0.15 and the pitching moment at zero lift is equal to 0.08, determine the trim lift coefficient. If the center of gravity of the airplane is located at \( x_{CG}/\bar{c} = 0.3 \), determine the stick-fixed neutral point.

Answer: \( C_{M} = 0.08 - 0.15 C_{L} = 0 \rightarrow C_{L_{\text{trim}}} = 0.53 \). Then, \( h_{n} = -0.15 = \frac{x_{CG} - x_{NP}}{\bar{c}} \rightarrow x_{NP}/\bar{c} = 0.45 \).

6. Problem 2.2 in Nelson [1], pp. 85: For the data shown in Fig. 2, determine the following:

- The stick fixed neutral point.
- If we wish to fly the airplane at a velocity of 125 ft/s at sea level, what would be the trim lift coefficient and what would be the elevator angle for trim?

![Figure 2:](image_url)

Answer: \( C_{M_{0 L}} = 0.04 \frac{dC_{M}}{dC_{L}} = -0.15 \), and \( C_{L_{a}} = -0.0075/\text{deg} \). Thus, \( \frac{x_{NP}}{\bar{c}} = 0.4 \). \( L = W \rightarrow C_{L} = 0.82 \). Trim: \( C_{M} = 0 \rightarrow \delta_{e} = -11.1^\circ \).

7. Reading for next lecture: No reading assignment. Next lecture will have practice problems on the concepts introduced so far.

References